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AN ECONOMIC ORDER QUANTITY MODELS FOR AMELIORATING ITEMS WITH QUADRATIC DEMAND AND PARTIAL BACKLOGGING RATE

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ABSTRACT
This study develops an Economic Order Quantity (EOQ) model for ameliorating inventory items subject to a time-dependent quadratic demand rate and partial backlogging. Amelioration, where the quality or quantity of stored items increases over time as observed in the items like poultry, fish, fruit and piggery that are incorporated into the model alongside demand patterns that accelerate quadratically. The model also allows for partial shortages, with backlogging rates dependent on customer waiting time. Using calculus-based methods, a cost function was formulated to determine the optimal cycle length that minimizes total variable costs. Numerical examples validate the model, and sensitivity analysis identifies the key parameters that influence optimal cycle length, total cost, and order quantity. The result shows that parameters like amelioration rate, cost components, and demand coefficients significantly influence the inventory decisions. This model offers a practical framework for inventory management in industries dealing with improving items and non-linear customer demand behavior.

Keywords: Economic Order Quantity, Amelioration, Quadratic Demand, Partial Backlogging, Inventory Management

INTRODUCTION

Ford W. Harris was the first to invent an Economic Order Quantity (EOQ) model in 1913; the Harris maiden model marked 112years by the year 2025, amazingly, the EOQ model continues to be the central model in the inventory management, supply chain and logistics management. Gwanda *et al*. (2023) explained that Amelioration on the other hand, refers to a situation where stocked items increase in quantity and/or quality while in stock. Some items have the property of incurring amelioration and deterioration though simultaneously as they are kept in warehouse. Gwanda et al. (2019a) developed an EOQ model for ameliorating items to describe the three stages and finally evolved a comprehensive model that merged all the models together to determine the optimal ordering quantity under the cost minimization. Numerical examples were given to illustrate the developed model and sensitivity analysis was carried out on the results obtained from one of the examples in order to see the effect of parameter changes on the decision variables. Gwanda *et al.* (2019b) reported that when unripe items like fruits were taken to warehouse; they stay dormant for sometimes without showing any sign of change. This period of full persisted for some time until when they became ripe and thus incurred amelioration in value and utility. An EOQ model for such items was formatted where the demand was constant throughout the cycle period. Srivastava (2017) developed an inventory model for ameliorating/deteriorating items with trapezoidal demand and complete backlogging under inflation and time discounting.

Srivastava (2017) proposed an inventory model for ameliorating/deteriorating items with inflationary condition and time discounting rate and also completely backlogged shortage. Vandana (2019) has carried out a research work titled, “A two-echelon inventory model for ameliorating /deteriorating items with single vendor and multi-buyers’’ where a model that proposes a fixed period for buyers and reduces the integrated total cost of the inventory was considered. The model discussed a case of single manufacturer who produces the ameliorating items and sells the finished goods to the multiple buyers. Zhang *et al.* (2022) proved the existence and uniqueness of the optimal order-up-to level, the reorder point and the preservation technology investment under any given two cases and presented an algorithm to search for decision variables such that the total profit per unit time is maximized. The linearity or otherwise of stock dependent demand was also a subject of intensive research. Cárdenas-Barrón (2020) for instance, studied an EOQ model with nonlinear stock dependent holding cost, nonlinear stock dependent demand and trade credit. The authors developed the model from retailers’ point of view where the supplier offers a trade credit period to the retailer.

The model relaxed the traditional assumption of zero ending inventory level to a non-zero ending which was consequently obtained as positive, zero or negative. Gwanda and Sani (2011) stated that demand for items that ameliorate would be expected to change over time, their expectations leads to many researches with different demand patterns including this research. Begum *et al*. (2012) reported that the scope of the model lies in its applicability in the management inventories of time-quadratic demand. It is also seen that large pile of goods displayed in a supermarket would motivate the customer to buy more. So the presence of inventory has a motivational effect on the people around it. There might be also occasional shortages in inventory due to many reasons. Therefore, they developed an EOQ model for the inventory of deteriorating items, taking demand rate and allowing shortages in inventory. In their research, they presented an inventory model for deteriorating items with quadratic demand shortages were allowed and partially backordered. The backlogging rate is a variable and dependent on the waiting time for the next replenishment. Khanra *et al*. (2010) considered an EOQ model with stock and price dependent demand rate. Sana and Chaudhuri (2004) developed production policy for a deteriorating item with time-dependent demand and shortages. Gwanda and Sani (2012) extended their earlier model of Gwanda and Sani (2011) and considered for linear trended in demand. Mamuda *et al.* (2025) developed models for an inventory items that are ameliorating with quadratic demand pattern where shortages was not allowed.

In this research work, it is intended to extend the work of Mamuda *et al.* (2025) to model the situation where the demand rate was quadratic in nature and the items ameliorate at a constant rate, were shortages are allowed. In addition, a new approach to the inventory was introduced by taking the demand rate to be the quadratic function of time where the model was partially backlogged. Mamuda *et al.* (2025) explained that, Quadratic function of time was seen to be the best representation of accelerated growth in the demand. Five solutions of the developed model were discussed and it was illustrated with the help of numerical examples. Sensitivity analysis of the optimal solution with respect to changes in different parameter values was also carried out.

Assumptions and Notations

The EOQ model for ameliorating items with quadratic demand and partial backlogging rate was developed based on the following assumptions and notations:

1. The inventory system involves only one single item and one stocking point
2. Amelioration occurs when the items are effectively in stock.
3. Shortages are allowed
4. Demand rate is a Quadratic function of time
5. The unit cost of an item is constant
6. The replenishment cost is constant per each replenishment
7. T: Cycle length

8. v_0: Initial inventory level
9. C_h: Inventory holding cost per cycle
10. D_R: Total demand rate within the cycle
11. β: Constant rate of Amelioration
12. A_T: Total Ameliorated amount within the cycle
13. T_Q: Total on Hand inventory within the cycle
14. C_1: Ordering Cost per unit
15. C_2: shortage cost per cycle due to backlog
16. C_3: opportunity cost per cycle due to lost sales
17. σ: Constant rate of Backorder
18. ρ: Constant Backlogging rate

MATERIAL AND METHOD

Model Formulation

In this research work, the calculus method was employed to solve the developed model by analysing the on Hand Inventory Items and find its solutions using relevant boundary conditions within the interval $(0 \leq t \leq t_1)$. The on Hand Inventory could then be solved to obtain the following totals: On Hand Inventory, Holding Cost, and Ameliorated amount within the cycle period. The Total Backlogged and Total Demand Unsatisfied Items during the cycle period could be used to solve the total variable cost within the interval $(t_1 \leq t \leq T)$. The concept of Maxima and Minima was also employed to minimize the Total Variable Cost (TVC) to obtain the cost function. The obtained Cost Function was solved using Microsoft Excel spreadsheet.

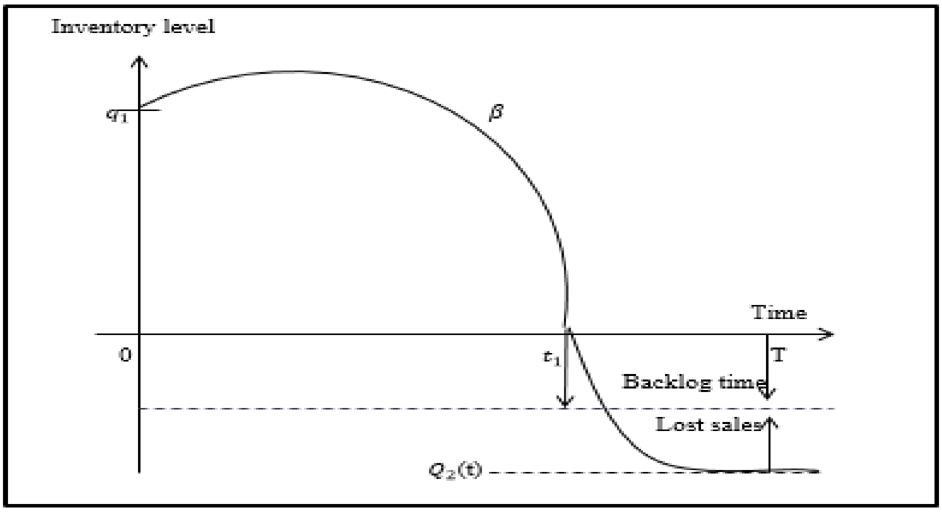

Fig. 1: The graphical representation for the inventory system

In the initial stage, within the interval $0 \le t \le T$, amelioration occurs at a constant rate of β and the demand was Quadratic in nature. The differential equation that describes the state of inventory level $V(t)$ is given by:

$$\frac{dV(t)}{dt} - \beta V(t) = -(at^2 + bt + c), \qquad 0 \le t \le T \tag{1}$$

with the boundary conditions given below:

at $t = 0,\ V(0) = v_0$ at $t = T,\ V(T) = 0$

By applying the integrating factor method to equation (1), $V(t)$ is obtained as:

$$V(t) = \frac{1}{\beta^3}[at^2\beta^2 + 2at\beta + 2a + bt\beta^2 + b\beta + c\beta^2] + ke^{\beta t} \tag{2}$$

Applying the first boundary condition: at at $t = 0; V(0) = v_0$, the value of constant k is:

$$v_0 = \frac{1}{\beta^3}[2a + b\beta + c\beta^2] + k \tag{3}$$

Equation (3) becomes:

$$k = v_0 - \frac{1}{\beta^3}[2a + b\beta + c\beta^2] \tag{4}$$

Substituting equation (4) into equation (2) to obtain:

$$V(t) = \frac{1}{\beta^3}[at^2\beta^2 + 2at\beta + 2a + bt\beta^2 + b\beta + c\beta^2] + \left[v_0 - \frac{1}{\beta^3}[2a + b\beta + c\beta^2]\right]e^{\beta t} \tag{5}$$

$$V(t) = \frac{1}{\beta^3}[at^2\beta^2 + 2at\beta + 2a + bt\beta^2 + b\beta + c\beta^2] - \frac{1}{\beta^3}[2a + b\beta + c\beta^2]e^{\beta t} + v_0e^{\beta t} \tag{6}$$

Applying the second boundary condition, at $t = T;\ V(T) = 0$ to obtain equation (7)

$$0 = \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2] - \frac{1}{\beta^3}[2a + b\beta + c\beta^2]e^{\beta T} + v_0e^{\beta T} \tag{7}$$

Equation (7) becomes:

$$v_0 = \frac{1}{\beta^3}[2a + b\beta + c\beta^2] - \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T} \tag{8}$$

Substituting equation (8) into equation (6) to obtain:

$$V(t) = \frac{1}{\beta^3}[at^2\beta^2 + 2at\beta + 2a + bt\beta^2 + b\beta + c\beta^2] - \frac{1}{\beta^3}[2a + b\beta + c\beta^2]e^{\beta t} + \left[\frac{1}{\beta^3}[2a + b\beta + c\beta^2] - \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T}\right]e^{\beta t} \quad (9)$$

Simplifying equation (9) we obtain:

$$V(t) = \frac{1}{\beta^3}[at^2\beta^2 + 2at\beta + 2a + bt\beta^2 + b\beta + c\beta^2] - \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]\left(e^{\beta(t-T)}\right) \quad (10)$$

Hence equation (10) is the expression for the on-hand inventory within the interval $(0 \le t \le T)$

The total demand rate within the interval $0 \le t \le T$ is given by:

$$D_R = \int_0^T (at^2 + bt + c)\,dt$$

$$D_R = \frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT \quad (11)$$

The total on Hand inventory within the interval $0 \le t \le T$ is given by:

$$T_Q = \int_0^T V(t)\,dt$$

$$T_Q = \int_0^T \left[\frac{1}{\beta^3}[at^2\beta^2 + 2at\beta + 2a + bt\beta^2 + b\beta + c\beta^2] - \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]\left(e^{\beta(t-T)}\right)\right]dt \quad (12)$$

Simplifying equation (12) we obtain:

$$T_Q = \frac{1}{\beta^3}\left[\frac{1}{3}aT^3\beta^2 + aT^2\beta + 2aT + \frac{1}{2}bT^2\beta^2 + bT\beta + cT\beta^2\right] - \frac{1}{\beta^4}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]\left(1 - e^{-\beta T}\right) \quad (13)$$

The Ameliorated amount over the interval $0 \le t \le T$ is given by

$$A_T = D_R - v_0$$

$$A_T = \frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT - \left[\frac{1}{\beta^3}[2a + b\beta + c\beta^2] - \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T}\right] \quad (14)$$

$$A_T = \frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT + \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T} - \frac{1}{\beta^3}[2a + b\beta + c\beta^2] \quad (15)$$

The holding inventory cost within the interval $0 \le t \le T$ is obtained as:

$$C_h = iCT_Q$$

$$C_h = \frac{iC}{\beta^3}\left[\frac{1}{3}aT^3\beta^2 + aT^2\beta + 2aT + \frac{1}{2}bT^2\beta^2 + bT\beta + cT\beta^2\right] - \frac{iC}{\beta^4}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]\left(1 - e^{-\beta T}\right) \quad (16)$$

The total amount backordered due to backlogging during the cycle time is given by:

$$B_T = \int_{t_1}^{T} -Q_2(t)\, dt$$

$$B_T = \int_{t_1}^{T} \sigma\rho(t - t_1)\, dt \quad (17)$$

$$= \frac{\sigma\rho}{2}[T^2 - 2t_1T + t_1^2\ ]$$

$$= \frac{\sigma\rho}{2}[T - t_1]^2 \quad (18)$$

Total amount of demand items unsatisfied during the cycle time is given by:

$$I_{lost} = \int_{t_1}^{T} \rho(1 - \sigma)\, dt$$

$$I_{lost} = [\rho(1 - \sigma)]_{t_1}^{T} \quad (19)$$

$$= \rho(1 - \sigma)T - \rho(1 - \sigma)t_1$$

$$= \rho(1 - \sigma)(T - t_1) \quad (20)$$

Total variable cost per unit time will be calculated as follows:

$TVC(T) = \frac{1}{T}$[Ordering cost + Holding cost per cycle-Amelioration cost per cycle + Shortage cost per cycle due to backlog + Opportunity cost due to lost sales]

$$\mathrm{TVC(T)} = \frac{1}{T}\begin{bmatrix} \left[C_1 + \frac{C_2}{2}\sigma\rho(T-t_1)^2 + C_3\rho(1-\sigma)(T-t_1)\right] + \\ \frac{iC}{\beta^4}\begin{bmatrix} \beta\left[\frac{1}{3}aT^3\beta^2 + aT^2\beta + 2aT + \frac{1}{2}bT^2\beta^2 + bT\beta + cT\beta^2\right] - \\ \left[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2\right]\left(1 - e^{-\beta T}\right) \end{bmatrix} - \\ C\begin{bmatrix} \frac{1}{\beta^3}\left[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2\right]e^{-\beta T} \\ + \left(\frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT\right) - \frac{1}{\beta^3}\left[2a + b\beta + c\beta^2\right] \end{bmatrix} \end{bmatrix} \tag{21}$$

$$\mathrm{TVC(T)} = \begin{bmatrix} \left[\frac{C_1}{T} + \frac{C_2}{2T}\sigma\rho(T-t_1)^2 + \frac{C_3}{T}\rho(1-\sigma)(T-t_1)\right] + \\ \frac{iC}{T\beta^3}\left[\frac{1}{3}aT^3\beta^2 + aT^2\beta + 2aT + \frac{1}{2}bT^2\beta^2 + bT\beta + cT\beta^2\right] - \\ \frac{iC}{T\beta^4}\left[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2\right]\left(1 - e^{-\beta T}\right) - \\ \frac{C}{T\beta^3}\left[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2\right]e^{-\beta T} \\ -\frac{C}{T}\left(\frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT\right) + \frac{C}{T\beta^3}\left[2a + b\beta + c\beta^2\right] \end{bmatrix} \tag{22}$$

To obtain the value of T which minimizes the total variable cost per unit time, equation (22) will be differentiated with respect to T as follows:

$$\frac{d}{dT}\left(\mathrm{TVC(T)}\right) = 0 \tag{23}$$

Differentiating equation (22) with respect to T, applying quotient rule the result follows:

$$\frac{d\mathrm{TVC(T)}}{dT} = \begin{bmatrix} -\frac{C_1}{T^2} + \frac{C_2\sigma\rho(T^2-t_1^2)}{2T^2} + \frac{C_3\rho(1-\sigma)t_1}{T^2} + \frac{iC}{\beta^2}\left(a + \frac{2}{3}aT^2\beta + \frac{1}{2}b\beta\right) \\ -\frac{iC}{T\beta^3}\left[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2\right]e^{-\beta T} \\ -\frac{iC}{T^2\beta^3}\left(aT^2\beta - \frac{2a}{\beta} - b - c\beta\right)\left(1 - e^{-\beta T}\right) \\ +\frac{C}{T\beta^2}\left(aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2\right)e^{-\beta T} \\ -\frac{C}{T^2\beta^3}\left[aT^2\beta^2 - 2a - b\beta - c\beta^2\right]e^{-\beta T} - C\left(\frac{2}{3}aT + \frac{1}{2}b\right) \\ -\frac{C}{T^2\beta^3}\left[2a + b\beta + c\beta^2\right] \end{bmatrix} \tag{24}$$

Setting equation (24) to zero, we obtain the optimal T which minimizes the total variable cost per unit time; hence we obtain equation (25).

$$\begin{bmatrix} -2\beta^3 C_1 + \beta^3 C_2 \sigma\rho(T^2 - t_1^2) + 2\beta^3 C_3 \rho(1-\sigma)t_1 + \\ 2iC\left(\frac{2}{3}aT^4\beta^2 + aT^2\beta + \frac{1}{2}bT^2\beta^2\right) - \\ 2iCT[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T} - \\ 2iC\left(aT^2\beta - \frac{2a}{\beta} - b - c\beta\right)\left(1 - e^{-\beta T}\right) - 2CT^2\beta^3\left(\frac{2}{3}aT + \frac{1}{2}b\right) \\ +CT\beta(aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2)e^{-\beta T} - \\ 2C[aT^2\beta^2 - 2a - b\beta - c\beta^2]e^{-\beta T} - 2C[2a + b\beta + c\beta^2] \end{bmatrix} = 0 \quad (25)$$

The Economic Order Quantity was derived from equation (8) for:

$$EOQ = v_0 = D_R - A_T$$

$$EOQ = \frac{1}{\beta^3}[2a + b\beta + c\beta^2] - \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T} \quad (26)$$

Equation (25) was solved to obtain the optimum values T^* of T using any suitable numerical method provided that, $\frac{d^2}{dT^2}\left(TVC(T^*)\right) > 0$ is true.

RESULT AND DISCUSSION

Numerical Examples

The table 1 below shows the solutions of ten different numerical examples with varying parameters. The obtained values presented in the last and second to the last column of table 1, titled EOQ and $TVC(T^*)$ the solutions computed from equation (26) and (25) respectively, while the remaining parameters value was arbitrarily chosen. In example 1, for instance, the assigned parameters' values are: $a = 300$, $b = 150$, $c = 100$, $\beta = 0.4$, $c_1 = 500$, $c_2 = 400$, $c_3 = 300$, $i = 0.3$, $C = 200$, and T^*=11days, $TVC(T)^* = 70087662$ has shown that with the values assigned to the decision parameters, the optimal ordering quantity is obtained as 551957units within an optimal cycle period of 11days.

Table 1: Computed EOQ obtained from numerical examples with varying parameters

a	b	c	β	ρ	σ	C_1	C_2	C_3	C	i	T^*	$TVC(T)^*$	EOQ^*
300	150	100	0.4	1	1	500	400	300	200	0.3	11days	70087662	551957units
300	150	100	0.4	0.9	0.8	500	400	300	200	0.4	6days	128517523	551955units
400	300	200	0.5	0.9	0.8	500	400	300	200	0.4	7days	83410175	952054units
450	350	300	0.55	0.7	0.6	150	200	250	200	0.5	5days	103781675	1034305units
200	150	100	0.5	0.1	0.2	150	100	50	250	0.5	4days	91236959	236026units

Sensitivity Analysis

Table 2: Sensitivity analysis on example one of table one to see the effect of parameter changes

Parameters	% change in parameter Value	Change in results		
		T^*	$TVC(T)^*$	EOQ^*
$\boldsymbol{a}$	+50	0.0274	111315828	828519
	+25	0.03014	85640369	690238
	−25	0.03014	54534956	413675
	−50	0.03014	38982250	275394
$\boldsymbol{b}$	+50	0.03014	73198203	832738
	+25	0.03014	71642933	692347
	−25	0.03014	68532392	411566
	−50	0.03014	66977122	271175
$\boldsymbol{c}$	+50	0.0274	70907245	551841
	+25	0.03014	70497454	551899
	−25	0.03014	69677871	552014
	−50	0.03014	69268080	552072
$\boldsymbol{\beta}$	+50	0.00000	0000000	000000
	+25	0.06849	16329756	354432
	−25	0.01918	252604542	975789
	−50	0.0137	1157061600	2170755
$\boldsymbol{\rho}$	+50	0.03014	70087662.43	551957
	+25	0.03014	70087662.42	551957
	−25	0.03014	70087662.39	551957
	−50	0.03014	70087662.37	551957
$\boldsymbol{\sigma}$	+50	0.03014	70087662.43	551957
	+25	0.03014	70087662.42	551957
	−25	0.03014	70087662.39	551957
	−50	0.03014	70087662.37	551957
$\boldsymbol{C_1}$	+50	0.03014	70095957.86	551957
	+25	0.03014	70091810.13	551957
	−25	0.03014	70083514.67	551957
	−50	0.0274	77090292.73	551957
$\boldsymbol{C_2}$	+50	0.03014	70087662.43	551957
	+25	0.03014	70087662.42	551957
	−25	0.03014	70087662.39	551957
	−50	0.03014	70087662.37	551957

C_3	$+50$	0.03014	70087662	551957
	$+25$	0.03014	70087662	551957
	-25	0.03014	70087662	551957
	-50	0.03014	70087662	551957
C	$+50$	0.01918	165218437	551955
	$+25$	0.0274	86734648.16	551956
	-25	0.03288	56215719.73	551957
	-50	0.06027	17516962	551960
i	$+50$	0.0137	154226133.7	551954.9
	$+25$	0.01644	128517518.5	551955.2
	-25	0.0	0.0	0.0
	-50	0.0	0.0	0.0

DISCUSSIONS

i- T^* Increase with increase in $a, C\ and\ i$ and decrease with decrease in $\beta\ and\ C_1$, while the remaining parameters remains non sensitive to the decision variable.

ii- $TVC(T)^*$ Increase with the increase in $\beta\ and\ C_1$, and decrease with decrease in $a, b, c, \rho, \sigma, C_2, C\ and\ i$, while the parameter C_3 is non-sensitive to changes in the decision variable.

iii- EOQ^* Increases with increase in $c, \beta, C\ and\ i$ and decrease with decrease in $a\ and\ b$, while the remaining parameters $\rho, \sigma, C_1, C_2\ and\ C_3$ remains non-sensitive to changes in the decision variable.

From the above analysis, it is clearly observed that in the optimal(T^*), only five parameters are sensitive to changes in the decision variables, while the remaining seven parameters are non-sensitive. In the optimal($TVC(T)^*$), ten parameters are sensitive to changes in the decision variables; while the remaining two parameters are non-sensitive to changes. In the optimal (EOQ^*), only six parameters are sensitive to changes in the decision variables, while the remaining six parameters remains non-sensitive to changes. But the amelioration rate β, increases with increase in $TVC(T)^*\ and\ EOQ^*$, while decrease with decrease in T^*. Also ($C\ and\ i$) increases with increase in two decision variables (T^* and EOQ^*) but decrease with decrease in the decision variable $TVC(T)^*$. This is understandable, because the decrease in the inventory cost and the inventory charges (i) could cause the increase in (EOQ^*), and the days to which the order will be completed. This is evident from the fact that when the $TVC(T)^*$ is high, the stockiest may buy a little items leading to shorter days to dispose the inventory, but in the case of this research, it is clearly observed that the decrease in $TVC(T)^*$, resulted in high (T^* and EOQ^*). With the low variable cost, the stockiest has the ability to purchase high inventory items that would help to fetch maximum profit.

CONCLUSIONS

In this article, EOQ model for Ameliorating Items with Quadratic Demand and Partial Backlogging rate was developed, in the research, the sensitivity of the decision variables T* (optimal cycle length), TVC(T)* (optimal total variable cost), and EOQ* (optimal economic order quantity) are strongly influenced by different sets of parameters. T* is affected by five key parameters: a, C, i (increase) and β, C_1 (decrease), indicating these are critical for determining the optimal timing of inventory replenishment. $TVC(T)$* is influenced by ten parameters, most significantly by β and C_1 (increase), and $a, b, c, \rho, \sigma, C_2, C$ and i (decrease), suggesting that it is the most sensitive decision variable to changes in the system dynamics. EOQ* increases with c, β, C *and* i and decreases with *and* b, while remaining insensitive to others, marking six parameters as influential. Additionally: The amelioration rate β has an inverse relationship with T* but a direct relationship with $TVC(T)$* and (EOQ)*. The parameters C and i increase both T* and (EOQ)* but decrease $TVC(T)$*, showing their dual effect on cost and inventory volume decisions. This implies that reducing variable costs (TVC) enables stockiest to hold larger inventories over longer periods (higher T* and (EOQ)*), potentially increasing profit margins. Conversely, a high $TVC(T)$* discourages large inventory orders and shortens the optimal ordering cycle.

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