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A MATHEMATICAL MODELLING FRAMEWORK FOR EPIDEMIC DYNAMICS UNDER QUARANTINE AND MEDIA-INDUCED AWARENESS USING AN SIQRM FRAMEWORK

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ABSTRACT
Dengue control remains challenging due to the complex interaction between disease transmission, intervention effectiveness, and human behavioural responses. This study develops a human-host SIQRM mathematical model incorporating quarantine and infection-induced media awareness to investigate their combined influence on epidemic dynamics. In this reduced human-host formulation, mosquito-mediated transmission is represented implicitly through the effective transmission process rather than through explicit mosquito compartments. The model consists of susceptible, infectious, quarantined, recovered, and media-awareness compartments and is analysed through positivity, boundedness, equilibrium analysis, reproduction number estimation, sensitivity analysis, and numerical simulations. The analytical results establish the conditions governing disease invasion and persistence, with the baseline parameter set yielding $R_0 > 1$, indicating the potential for epidemic growth. Numerical simulations over 365 days show that media awareness reduces epidemic burden. Without media feedback, the infectious population reached approximately 1726.30 individuals, whereas the inclusion of media awareness reduced the peak infectious population to 670.88 individuals, corresponding to a 61.14% reduction. Furthermore, cumulative infections decreased from 19,618.10 to 12,321.56, representing a 37.19% reduction. Increasing media-generation intensity further reduced epidemic magnitude and delayed the infection peak. Sensitivity analysis revealed that transmission intensity and residual infectiousness during quarantine positively influenced epidemic persistence, whereas quarantine and recovery rates contributed to transmission reduction. The results indicate that quarantine effectiveness depends strongly on limiting infectious contact after isolation, while media awareness provides an additional mechanism for reducing epidemic severity. The proposed framework highlights the importance of integrating behavioural-information dynamics with conventional intervention strategies for improved epidemic control.

Keywords: Dengue modelling; SIQRM model; Lyapunov stability; Disease-free equilibrium; Endemic equilibrium; Nonlinear epidemic systems.

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1.Introduction

Dengue continues to pose a considerable public-health challenge as a vector-borne disease in tropical and subtropical regions, where transmission is influenced by interactions among human hosts, *Aedes aegypti* populations, environmental conditions, and disease-control measures. Mathematical modelling has become an important tool for gaining insight into infectious-disease dynamics because it allows the mechanisms of transmission to be analysed, influential parameters to be identified, and the potential effects of interventions to be evaluated. Classical compartmental models provide a useful foundation for epidemic analysis, but they generally assume relatively fixed behavioural patterns and may therefore be limited in representing adaptive human responses during disease outbreaks.

Although dengue transmission is biologically mediated by mosquito vectors, the present study adopts a reduced human-host formulation rather than an explicit human-vector model. The objective is to investigate the interaction between quarantine and media-induced behavioural awareness within the human population. Accordingly, the effect of mosquito-mediated exposure is represented implicitly through the effective transmission process, while mosquito population dynamics are not modelled as separate compartments. This simplification enables the analysis to remain focused on the intervention and behavioural mechanisms considered in the SIQRM-media framework and should therefore be interpreted as a reduced representation of dengue transmission rather than a complete host-vector description.

In recent years, epidemic modelling has increasingly demonstrated the importance of human behaviour and information availability in influencing disease spread. Awareness, risk perception, and behavioural adaptation can alter contact patterns and thereby reduce opportunities for transmission. Funk et al. (2010) emphasised that behavioural components should be incorporated into epidemic models because changes in public response may influence epidemic progression. Similarly, Kiss et al. (2010) showed that information transmission can be dynamically coupled with disease spread, resulting in an interacting information-epidemic system.

Mathematical models that incorporate media awareness have also demonstrated that information campaigns may influence epidemic outcomes by encouraging protective behaviour. Misra et al. (2011) introduced an epidemic modelling framework with awareness and showed that disease prevalence can be reduced when media-generated information induces changes in population behaviour. Samanta et al. (2013) subsequently showed that awareness programmes can influence epidemic thresholds and equilibrium behaviour. More recent studies have treated awareness as a dynamic process in which behavioural feedback and persistence may also influence information dynamics and disease transmission (Agaba et al., 2017).

Despite these developments, many existing models consider information effects separately from epidemiological interventions. Awareness-based models have improved the representation of behavioural responses, whereas quarantine-based models provide useful insights into reducing infectious contact but often assume relatively constant compliance with intervention measures. In practice, the effectiveness of quarantine may depend not only on implementation but also on public awareness, perceived risk, and behavioural response. An integrated framework combining quarantine dynamics, behavioural awareness, and media feedback can therefore provide a more suitable basis for examining the interaction between epidemiological intervention and information-driven behaviour.

In this study, an SIQRM-media model is developed to investigate the combined effects of quarantine and media awareness on epidemic dynamics. The framework consists of susceptible, infectious, quarantined, and recovered human compartments together with a media-awareness

variable, and it examines the interaction between epidemiological progression and information-driven behavioural response.

Aim and Objectives of the Study

i. To develop an SIQRM-media mathematical model incorporating quarantine dynamics and media-induced behavioural awareness.
ii. To analyse the mathematical properties of the model, including positivity, boundedness, equilibrium conditions, and the basic reproduction number.
iii. To assess the effects of epidemiological, quarantine, and awareness-related parameters through sensitivity analysis.
iv. To examine the effectiveness of quarantine and media-awareness measures using numerical simulations.

2. Literature Review

2.1 Classical Epidemic Modelling and Compartmental Frameworks

Mathematical epidemic models provide a systematic approach for understanding infectious disease transmission by representing population groups according to their epidemiological status. The foundation of compartmental modelling was established by Kermack and McKendrick (1927), whose susceptible-infected-recovered (SIR) framework demonstrated that epidemic outbreaks depend on interactions between transmission processes and population susceptibility. This classical approach remains widely used because of its analytical simplicity and ability to provide important indicators such as epidemic thresholds and equilibrium behaviour. However, traditional compartmental models generally assume that transmission patterns remain unchanged during an outbreak and therefore provide limited representation of adaptive human behaviour.

Subsequent developments expanded classical models by incorporating demographic processes, disease-related mortality, and additional epidemiological compartments. Hethcote (2000) provided a comprehensive analysis of infectious disease models and demonstrated the importance of mathematical approaches for evaluating disease persistence and control strategies. Similarly, Brauer (2008) highlighted that compartmental models can be extended according to the biological characteristics of individual diseases. Despite these advances, many classical frameworks primarily focus on biological transmission mechanisms and often treat human behaviour as a fixed component rather than a dynamically changing factor.

For dengue and other infectious diseases where prevention depends strongly on public response, this limitation becomes important. Disease progression is not determined only by biological factors but may also be influenced by changes in awareness, protective behaviour, and acceptance of intervention measures. Therefore, modern epidemic modelling has increasingly incorporated additional mechanisms that represent behavioural adaptation and intervention responses.

2.2 Quarantine-Based Epidemic Models

Quarantine and isolation strategies have been widely incorporated into epidemic models to investigate how reducing infectious contact influences disease transmission. Unlike classical SIR-type models, quarantine-based frameworks introduce additional compartments to represent individuals who are separated from the general population due to infection status or exposure risk. These extensions allow researchers to evaluate the effectiveness of intervention strategies and identify conditions required for epidemic control.

Hethcote et al. (2002) examined quarantine effects in epidemic models and demonstrated that isolation measures can alter transmission dynamics by reducing interaction between infectious and susceptible individuals. However, such approaches generally assume that quarantine

effectiveness depends mainly on implementation rates and biological parameters. In reality, quarantine outcomes may also depend on compliance, public perception, and behavioural response.

This limitation has motivated further development of intervention models that consider the interaction between disease control measures and population behaviour. While quarantine-focused models provide important insights into reducing infectious contact, they often do not explicitly describe how individuals respond to information regarding disease risk. Consequently, quarantine alone may not fully represent outbreak dynamics where public awareness influences preventive behaviour and intervention acceptance.

The present study builds on this concept by incorporating quarantine dynamics together with media-induced awareness, allowing investigation of how information feedback may influence the effectiveness of quarantine-based control.

2.3 Awareness and Media-Induced Behavioural Epidemic Models

Increasing evidence indicates that human behaviour can substantially influence epidemic outcomes. Unlike traditional models where contact patterns remain constant, awareness-based models consider behavioural changes resulting from public information, perceived infection risk, and preventive actions. Funk et al. (2010) and Wang et al. (2019) reviewed the role of behavioural responses in infectious disease modelling and demonstrated that changes in human behaviour may modify transmission pathways.

Early awareness models generally represented information as an external factor influencing disease transmission. For example, Misra et al. (2011) and Sontag et al. (2021) introduced awareness mechanisms through media influence and showed that increased awareness could reduce disease spread by encouraging protective behaviour. However, their framework mainly considered awareness as a media-driven process rather than a variable dynamically affected by epidemic progression. Similarly, Samanta et al. (2013) and Al-Dmour et al. (2020) demonstrated that awareness programmes could influence epidemic outcomes, although the interaction between awareness evolution and intervention measures remained limited.

Later studies improved this representation by considering awareness as a dynamic process. Kiss et al. (2010) showed that information transmission can interact with epidemic progression, creating feedback between disease prevalence and public response. Agaba et al. (2017) further developed awareness-based modelling by incorporating behavioural feedback mechanisms, demonstrating that awareness dynamics can influence disease persistence and control.

Although these studies significantly improved the representation of behavioural effects, many awareness models primarily focused on information-driven behavioural change without explicitly considering intervention processes such as quarantine. Therefore, a remaining challenge is to understand how awareness and formal control strategies interact within the same modelling framework

2.4 Integrated Epidemic Intervention and Behavioural Models

Recent epidemic modelling approaches have increasingly recognised that effective disease control requires consideration of both biological interventions and behavioural responses. Media coverage, information availability, and public perception may influence compliance with control measures, while intervention policies may also affect awareness levels and behavioural adaptation.

Al Basir et al. (2018) demonstrated that media coverage and information delay can influence epidemic outcomes, highlighting the importance of incorporating communication processes into disease models. Similarly, Greenhalgh et al. (2015) showed that awareness programmes with delayed effects can modify epidemic behaviour and alter intervention outcomes.

These studies indicate that information dynamics should not be considered separately from epidemiological processes.

However, many existing frameworks still analyse awareness and intervention mechanisms independently. Quarantine models generally emphasise reduction of infectious contact, whereas awareness models focus on behavioural modification. The interaction between these two processes remains an important area for investigation because public awareness may determine compliance with quarantine measures and influence the overall effectiveness of intervention strategies.

Based on these considerations, the present study develops an SIQRM-media model that integrates quarantine dynamics with media-induced awareness within a unified human-host framework. Unlike models that consider intervention or awareness mechanisms separately, the proposed approach examines how behavioural information feedback modifies epidemic transmission under quarantine conditions. The framework therefore provides an analytical basis for evaluating the combined influence of quarantine effectiveness and awareness dynamics on epidemic control.

Existing mathematical models have established the importance of compartmental approaches, quarantine strategies, and awareness mechanisms in analysing infectious disease dynamics. Nevertheless, previous studies have frequently examined these components separately, limiting understanding of how behavioural information and intervention processes interact during epidemic progression. The present study addresses this gap by incorporating quarantine and media-awareness dynamics into a single SIQRM framework and analysing their combined influence through mathematical analysis and numerical simulations.

3.Methodology

3.1 Mathematical Model Formulation

To capture the dynamics of disease transmission in the presence of imperfect quarantine and media-induced awareness, a deterministic SIQRM model is developed. The human population is divided into five epidemiological compartments: susceptible individuals $S(t)$, infectious individuals $I(t)$, quarantined individuals $Q(t)$, and recovered individuals $R(t)$, while $M(t)$represents a dimensionless media-awareness index. The total human population is therefore expressed as

$$N(t) = S(t) + I(t) + Q(t) + R(t).$$

In this model, quarantine is assumed to be imperfect, meaning that individuals in the quarantined compartment may retain a reduced capacity for disease transmission. This assumption is represented by the parameter η, which describes the relative infectiousness of quarantined individuals compared with infectious individuals. Such a leaky-quarantine approach reflects realistic situations where complete isolation may not always be achieved due to incomplete compliance, delayed isolation, household exposure, or unavoidable contact during quarantine periods (Hethcote et al., 2002).

$$N(t) = S(t) + I(t) + Q(t) + R(t). \qquad (1)$$

Quarantined individuals retain a fraction $0 \leq \eta \leq 1$of the infectiousness of unquarantined individuals. Media awareness reduces effective transmission through

$$\Phi(M) = \frac{1}{1+M}, \qquad (2)$$

giving the force of infection

$$\lambda(t) = \frac{\beta(I + \eta Q)}{1 + M}. \qquad (3)$$

The governing system is

$$\frac{dS}{dt} = \Lambda - \frac{\beta S(I + \eta Q)}{1 + M} - \mu S, \qquad (4)$$

$$\frac{dI}{dt} = \frac{\beta S(I + \eta Q)}{1 + M} - (\gamma + \mu + \delta)I, \qquad (5)$$

$$\frac{dQ}{dt} = \gamma I - (\omega + \mu)Q, \qquad (6)$$

$$\frac{dR}{dt} = \omega Q - \mu R, \qquad (7)$$

and

$$\frac{dM}{dt} = \alpha_1 I - \alpha_2 M. \qquad (8)$$

In this, Λ represents the recruitment rate into the susceptible class, β is the transmission coefficient, η represent residual infectiousness while in quarantine, μ represents natural mortality rate, γ represents the rate of quarantine, δ represents the disease-induced mortality rate, ω represents the recovery rate of quarantined individuals, α_1 represents the media-generation rate due to infection, and finally $\alpha_2$represents the media-decay rate.

Table 1. State variables of the SIQRM model

Variable	Description	Unit
$S(t)$	Susceptible individuals	Individuals
$I(t)$	Infectious individuals	Individuals
$Q(t)$	Quarantined infectious individuals	Individuals
$R(t)$	Recovered individuals	Individuals
$M(t)$	Media-awareness index	Dimensionless

The total human population $N(t)$is not considered an independent state variable; it is a derived quantity calculated as $N(t) = S(t) + I(t) + Q(t) + R(t)$."

3.2 Parameter Values and Initial Conditions

Summary of the parameter used for the numerical simulation is provided in Table 2. Wherever possible parameters were those used in previous modelling studies, or were formulated from the model.

Table 2. Baseline parameter values used in numerical simulations

Parameter	Description	Value	Unit	Source
Λ	Recruitment rate	0.9973	$individuals^{-1}$ day^{-1}	Model assumption
β	Transmission coefficient	3.5194×10^{-6}	$individual^{-1}$ day^{-1}	Calculated from Eq. (9)
μ	Natural mortality rate	0.0000351	day^{-1}	Sahu and Dhar (2015)
γ	Quarantine rate of infectious individuals	0.050619	day^{-1}	Sahu and Dhar (2015)

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η	Relative infectiousness of quarantined individuals	0.50	Dimensionless	Sahu and Dhar (2015)
δ	Disease-induced mortality rate	0.04227	day^{-1}	Sahu and Dhar (2015)
ω	Recovery rate of quarantined individuals	0.042553	day^{-1}	Sahu and Dhar (2015)
α_1	Media-awareness generation rate	1.22×10^{-4}	day^{-1}	Calculated from Eq. (11)
α_2	Media-awareness decay rate	0.2535	day^{-1}	Zhou et al. (2019)

The transmission coefficient is rescaled from $\tilde{\beta} = 0.10\text{day}^{-1}$ as

$$\beta = \frac{\tilde{\beta}}{N_0} = \frac{0.10}{28414} = 3.5194 \times 10^{-6}. \quad (9)$$

For the media component,

$$M(0) = 0.0122(8) = 0.0976, \quad (10)$$

and

$$\alpha_1 = 0.0122(0.01) = 1.22 \times 10^{-4}. \quad (11)$$

Table 3. Initial conditions for numerical simulation

Variable	Initial value	Source
$S(0)$	28,410	Zhou et al. (2019)
$I(0)$	4	Zhou et al. (2019)
$Q(0)$	0	Assumed
$R(0)$	0	Zhou et al. (2019)
$M(0)$	0.0976	Derived from Zhou et al. (2019)
N_0	28,414	Derived

Positivity and Boundedness

Lemma 1. Positivity of Solutions

Let

$$(\&(S(0), I(0), Q(0), R(0), M(0)) \geq 0 \quad (12))$$

Then every solution of system (4) – (8) remains non-negative for all $t \geq 0$.

Proof

On the boundary $S = 0$,

$$\left.\frac{dS}{dt}\right|_{S=0} = \Lambda > 0. \quad (13)$$

For $I = 0$,

$$\left.\frac{dI}{dt}\right|_{I=0} = \frac{\beta\eta SQ}{1+M} \geq 0. \quad (14)$$

Similarly,

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$$\left.\frac{dQ}{dt}\right|_{Q=0} = \gamma I \geq 0, \qquad (15)$$

$$\left.\frac{dR}{dt}\right|_{R=0} = \omega Q \geq 0, \qquad (16)$$

and

$$\left.\frac{dM}{dt}\right|_{M=0} = \alpha_1 I \geq 0. \qquad (17)$$

Therefore, the vector field is inward pointing or tangent to every coordinate boundary, and $\mathbb{R}^5_+$is positively invariant.

Theorem 1.

All non-negative solutions of system (4)–(8) are bounded.

Proof

Adding equations (4)–(7) gives

$$\frac{dN}{dt} = \Lambda - \mu N - \delta I. \qquad (18)$$

Since $\delta I \geq 0$,

$$\frac{dN}{dt} \leq \Lambda - \mu N. \qquad (19)$$

By the comparison theorem,

$$N(t) \leq N(0)e^{-\mu t} + \frac{\Lambda}{\mu}(1 - e^{-\mu t}). \qquad (20)$$

Hence,

$$\limsup_{t\to\infty} N(t) \leq \frac{\Lambda}{\mu}. \qquad (21)$$

Since $I(t) \leq N(t)$,

$$\frac{dM}{dt} = \alpha_1 I - \alpha_2 M \leq \frac{\alpha_1 \Lambda}{\mu} - \alpha_2 M. \qquad (22)$$

Consequently,

$$\limsup_{t\to\infty} M(t) \leq \frac{\alpha_1 \Lambda}{\alpha_2 \mu}. \qquad (23)$$

Thus, the biologically feasible region is

$$\Omega = \left\{(S, I, Q, R, M), \in \mathbb{R}^5_+ : N \leq \frac{\Lambda}{\mu}\ M \leq \frac{\alpha_1 \Lambda}{\alpha_2 \mu}\right\}. \qquad (24)$$

Hence, Ωis positively invariant and all epidemiologically feasible solutions remain bounded.

Disease-Free Equilibrium

At the disease-free equilibrium,

$$I_0 = Q_0 = R_0 = M_0 = 0. \qquad (25)$$

The susceptible equation gives

$$0 = \Lambda - \mu S_0,$$

and therefore

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$$S_0 = \frac{\Lambda}{\mu}. \qquad (26)$$

Hence, the disease-free equilibrium is

$$\boxed{\mathcal{E}_0 = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right).} \qquad (27)$$

Basic Reproduction Number $\mathcal{R}_0$

The basic reproduction number is obtained using the next-generation matrix method (Diekmann et al., 1990; van den Driessche and Watmough, 2002). Take into account the infected-state vector.

$$x = \begin{pmatrix} I \\ Q \end{pmatrix}. \qquad (28)$$

Define

$$A = \gamma + \mu + \delta, \qquad (29)$$

and

$$D = \omega + \mu. \qquad (30)$$

The new-infection vector is

$$\mathcal{F} = \begin{pmatrix} \frac{\beta S(I + \eta Q)}{1 + M} \\ 0 \end{pmatrix}, \qquad (31)$$

while the transition vector is

$$\mathcal{V} = \begin{pmatrix} AI \\ -\gamma I + DQ \end{pmatrix}. \qquad (32)$$

Evaluating their Jacobians at $\mathcal{E}_0$gives

$$F = \begin{pmatrix} \frac{\beta\Lambda}{\mu} & \frac{\beta\eta\Lambda}{\mu} \\ 0 & 0 \end{pmatrix}, \qquad (33)$$

and

$$V = \begin{pmatrix} A & 0 \\ -\gamma & D \end{pmatrix}. \qquad (34)$$

The inverse of $V$is

$$V^{-1} = \begin{pmatrix} \frac{1}{A} & 0 \\ \frac{\gamma}{AD} & \frac{1}{D} \end{pmatrix}. \qquad (35)$$

Therefore,

$$FV^{-1} = \begin{pmatrix} \frac{\beta\Lambda}{\mu A} + \frac{\beta\eta\gamma\Lambda}{\mu AD} & \frac{\beta\eta\Lambda}{\mu D} \\ 0 & 0 \end{pmatrix}. \qquad (36)$$

Thus,

$$\mathcal{R}_0 = \rho(FV^{-1}), \qquad (37)$$

and hence

$$\boxed{\mathcal{R}_0 = \frac{\beta\Lambda}{\mu(\gamma + \mu + \delta)}\left(1 + \frac{\eta\gamma}{\omega + \mu}\right).} \qquad (38)$$

The reproduction number may be decomposed as

$$\mathcal{R}_0 = \mathcal{R}_I + \mathcal{R}_Q, \qquad (39)$$

where

$$\mathcal{R}_I = \frac{\beta\Lambda}{\mu(\gamma+\mu+\delta)} \qquad (40)$$

represents transmission before quarantine, and

$$\mathcal{R}_Q = \frac{\beta\eta\gamma\Lambda}{\mu(\gamma+\mu+\delta)(\omega+\mu)} \qquad (41)$$

represents residual transmission during quarantine.
Using the baseline parameter values,

$$\boxed{\mathcal{R}_0 = 1.7157.} \qquad (42)$$

Therefore, the baseline parameter set permits disease invasion.

Local Stability of the Disease-Free Equilibrium

Theorem 2

The disease-free equilibrium $\mathcal{E}_0$is locally asymptotically stable if

$$\mathcal{R}_0 < 1, \qquad (43)$$

and unstable if

$$\mathcal{R}_0 > 1. \qquad (44)$$

Proof

Three eigenvalues of the Jacobian evaluated at $\mathcal{E}_0$are

$$\lambda_1 = -\mu,\ \lambda_2 = -\mu,\ \lambda_3 = -\alpha_2. \qquad (45)$$

The remaining eigenvalues are obtained from

$$J_{IQ} = \begin{pmatrix} \frac{\beta\Lambda}{\mu} - A & \frac{\beta\eta\Lambda}{\mu} \\ \gamma & -D \end{pmatrix}. \qquad (46)$$

Its determinant is

$$\det\left(J_{IQ}\right) = D\left(A - \frac{\beta\Lambda}{\mu}\right) - \frac{\beta\eta\gamma\Lambda}{\mu}. \qquad (47)$$

Using equation (38),

$$\boxed{\det\left(J_{IQ}\right) = AD(1-\mathcal{R}_0).} \qquad (48)$$

Thus,

$$\det\left(J_{IQ}\right) > 0$$

when $\mathcal{R}_0 < 1$. Furthermore,

$$\mathrm{tr}\left(J_{IQ}\right) = \frac{\beta\Lambda}{\mu} - A - D. \qquad (49)$$

For $\mathcal{R}_0 < 1$,

$$\frac{\beta\Lambda}{\mu} < A,$$

which implies

$$\mathrm{tr}\left(J_{IQ}\right) < 0. \qquad (50)$$

Hence, both eigenvalues of J_{IQ}have negative real parts. Therefore, $\mathcal{E}_0$is locally asymptotically stable when $\mathcal{R}_0 < 1$. If $\mathcal{R}_0 > 1$, then $\det(J_{IQ}) < 0$, implying instability. □

Endemic Equilibrium and Its Existence

Let

$$\mathcal{E}^* = (S^*, I^*, Q^*, R^*, M^*) \qquad (51)$$

denote an endemic equilibrium with $I^* > 0$. Define

$$c = \frac{\alpha_1}{\alpha_2}, \qquad (52)$$

and

$$k = 1 + \frac{\eta\gamma}{\omega + \mu}. \qquad (53)$$

At equilibrium,

$$Q^* = \frac{\gamma}{\omega + \mu} I^*, \qquad (54)$$

$$R^* = \frac{\omega\gamma}{\mu(\omega + \mu)} I^*, \qquad (55)$$

and

$$M^* = \frac{\alpha_1}{\alpha_2} I^* = cI^*. \qquad (56)$$

Moreover,

$$I^* + \eta Q^* = \left(1 + \frac{\eta\gamma}{\omega + \mu}\right) I^* = kI^*. \qquad (57)$$

For $I^* > 0$, the infectious equilibrium equation gives

$$\frac{\beta S^* k}{1 + cI^*} = \gamma + \mu + \delta. \qquad (58)$$

Hence,

$$S^* = \frac{(\gamma + \mu + \delta)(1 + cI^*)}{\beta k}. \qquad (59)$$

The susceptible equilibrium equation gives

$$S^* = \frac{\Lambda - (\gamma + \mu + \delta)I^*}{\mu}. \qquad (60)$$

Equating equations (59) and (60),

$$\frac{(\gamma + \mu + \delta)(1 + cI^*)}{\beta k} = \frac{\Lambda - (\gamma + \mu + \delta)I^*}{\mu}. \qquad (61)$$

Since

$$\mathcal{R}_0 = \frac{\beta k \Lambda}{\mu(\gamma + \mu + \delta)}, \qquad (62)$$

equation (61) yields

$$1 + cI^* = \mathcal{R}_0 - \frac{\beta k}{\mu} I^*. \qquad (63)$$

Therefore,

$$\boxed{I^* = \frac{\mu(\mathcal{R}_0 - 1)}{\mu c + \beta k}}. \qquad (64)$$

Proposition 1

A unique biologically feasible endemic equilibrium exists if and only if

$$\mathcal{R}_0 > 1. \qquad (65)$$

Proof

Since

$$\mu c + \beta k > 0, \qquad (66)$$

equation (64) gives

$$I^* > 0 \iff \mathcal{R}_0 > 1. \qquad (67)$$

The remaining equilibrium variables are uniquely determined from equations (54)–(60). Therefore, a unique positive endemic equilibrium exists whenever $\mathcal{R}_0 > 1$. □

For the baseline parameter set,

$$\boxed{\mathcal{E}^* \approx (16596.88,\ 4.464,\ 5.305\ 6431.89\ 0.00215).} \qquad (68)$$

Numerical Stability of the Endemic Equilibrium

Because the nonlinear media-feedback term produces a high-order characteristic equation, local stability of the endemic equilibrium is assessed numerically.

Let

$$\Phi^* = \frac{1}{1 + M^*}, \qquad (69)$$

and

$$\Psi^* = \frac{1}{(1 + M^*)^2}. \qquad (70)$$

The coupled (S, I, Q, M)Jacobian block is

$$J_c(\mathcal{E}^*) = \begin{pmatrix} -\beta\Phi^*(I^* + \eta Q^*) - \mu & -\beta\Phi^* S^* & -\beta\eta\Phi^* S^* & \beta\Psi^* S^*(I^* + \eta Q^*) \\ \beta\Phi^*(I^* + \eta Q^*) & \beta\Phi^* S^* - (\gamma + \mu + \delta) & \beta\eta\Phi^* S^* & -\beta\Psi^* S^*(I^* + \eta Q^*) \\ 0 & \gamma & -(\omega + \mu) & 0 \\ 0 & \alpha_1 & 0 & -\alpha_2 \end{pmatrix}. \qquad (71)$$

The recovered compartment contributes

$$\lambda_R = -\mu. \qquad (72)$$

For the baseline parameter set, the numerical eigenvalues are approximately

$$\lambda_1 = -0.07735, \qquad (73)$$

$$\lambda_2 = -0.25326, \qquad (74)$$

$$\lambda_{3,4} = -8.607 \times 10^{-5} \pm 1.130 \times 10^{-3} i, \qquad (75)$$

and

$$\lambda_5 = -3.510 \times 10^{-5}. \qquad (76)$$

Thus,

$$\boxed{\max_i \Re(\lambda_i) = -3.510 \times 10^{-5} < 0.} \qquad (77)$$

Therefore, the endemic equilibrium is locally asymptotically stable for the baseline parameter set.

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4.Sensitivity Analysis

The normalized forward sensitivity index of the basic reproduction number R_0with respect to a parameter pis defined as:

$$\Upsilon_p^{R_0} = \frac{\partial R_0}{\partial p} \times \frac{p}{R_0}$$

where pre stands for any model parameter that affects the transmission dynamics. This is an index of the relative change in the value of R_0result when the parameter p is changed proportionately. If the sensitivity index is positive, the increase of the parameter will result in an increase of $R_0$and if the sensitivity index is negative the increase of the parameter will result in a decrease of R_0The responses of the basic reproduction number to each of the key parameters for the proposed SIQRM-media model are computed as the sensitivity indices:

$$\Upsilon_\beta^{R_0} = \frac{\partial R_0}{\partial \beta} \times \frac{\beta}{R_0}$$
$$\Upsilon_\eta^{R_0} = \frac{\partial R_0}{\partial \eta} \times \frac{\eta}{R_0}$$
$$\Upsilon_q^{R_0} = \frac{\partial R_0}{\partial q} \times \frac{q}{R_0}$$
$$\Upsilon_{\gamma_q}^{R_0} = \frac{\partial R_0}{\partial \gamma_q} \times \frac{\gamma_q}{R_0}$$
$$\Upsilon_\delta^{R_0} = \frac{\partial R_0}{\partial \delta} \times \frac{\delta}{R_0}$$

and

$$\Upsilon_\alpha^{R_0} = \frac{\partial R_0}{\partial \alpha} \times \frac{\alpha}{R_0}$$

where β, η, q, γ_q, δ, and αrepresent the transmission coefficient, relative infectiousness during quarantine, quarantine rate, recovery rate of quarantined individuals, disease-induced mortality rate, and media-awareness generation rate, respectively.

Because the media-awareness variable is not present in the disease-free equilibrium (M=0), the sensitivity of the parameters related to media-awareness is assessed according to their indirect effects on reducing transmission. The outcome of the sensitivity indices would give insight into which parameters play most important role in the persistence of the epidemics, and thus guide the prioritisation of effective control measures

Table 4. Normalized sensitivity indices at baseline

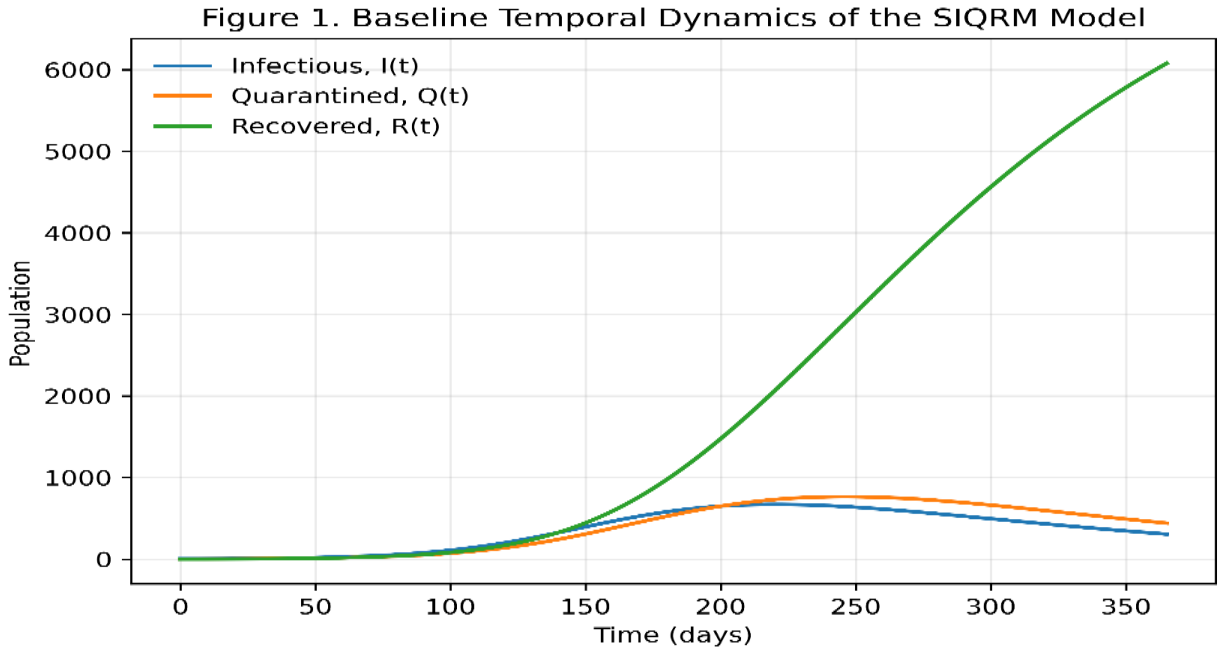
Parameter	Sensitivity index
μ	-1.0007
β	$+1.0000$
Λ	$+1.0000$
δ	-0.4549
η	$+0.3728$
ω	-0.3725
γ	-0.1720
α_1	0
α_2	0

The zero sensitivity indices obtained for the media-awareness parameters α_1 and α_2 require careful interpretation. These parameters do not directly influence the basic reproduction number R_0 because R_0 is evaluated at the disease-free equilibrium, where the media-awareness variable $M(t)$ is absent from the infected subsystem used in the next-generation matrix formulation. Therefore, the zero sensitivity values indicate that media parameters do not affect the initial invasion threshold rather than suggesting that media awareness has no role in epidemic control. In contrast, the numerical simulations consider the complete nonlinear system after disease introduction, where $M(t)$ evolves dynamically and influences transmission behaviour. Hence, the sensitivity analysis and numerical simulations describe different aspects of epidemic dynamics: R_0-based sensitivity identifies parameters affecting the initial establishment of infection, whereas simulations demonstrate the subsequent impact of media awareness on outbreak progression and infection reduction.

5.Numerical Simulation

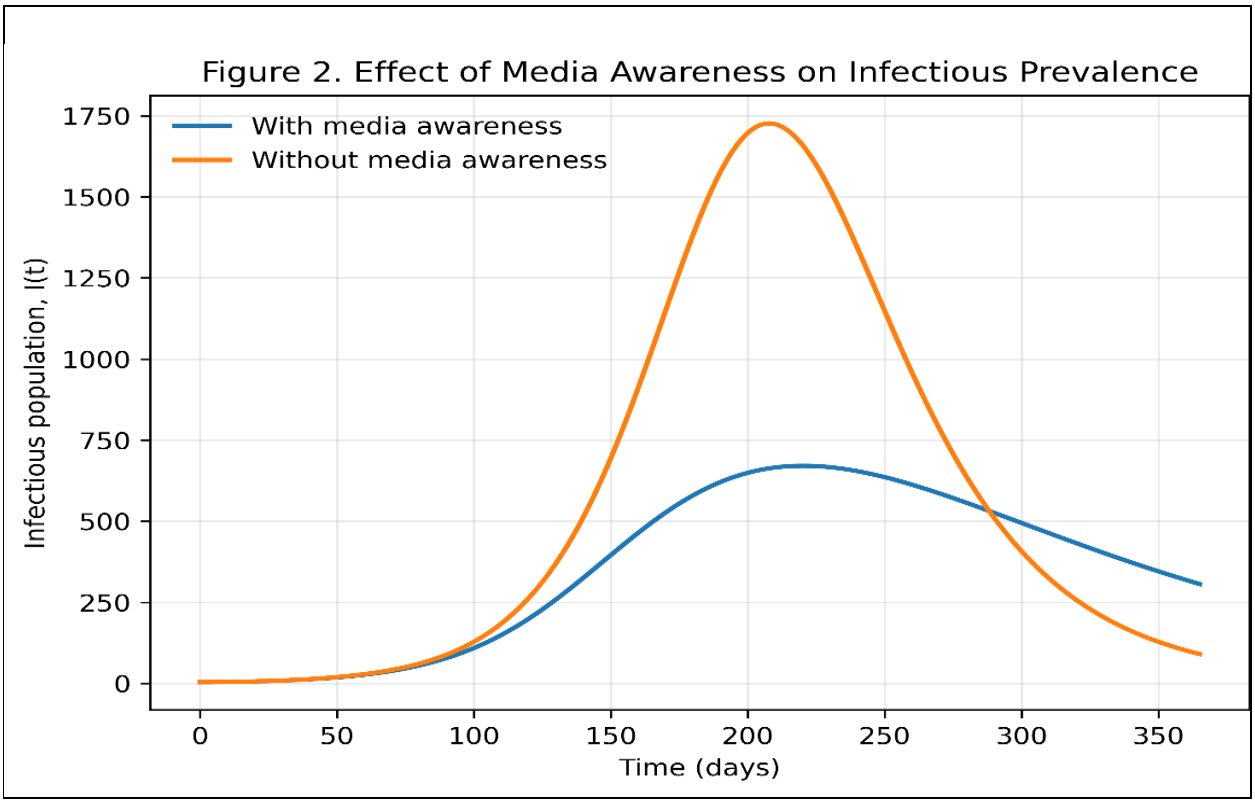
The SIQRM-media model equations have been numerically solved over a 365 day period with the aid of the adaptive Runge Kutta method in MATLAB with the baseline parameter values provided in Table 2 and initial conditions set in Table 3. Before the outbreak, there is baseline epidemic dynamics that must also be considered.

Figure 1: Baseline epidemic dynamics



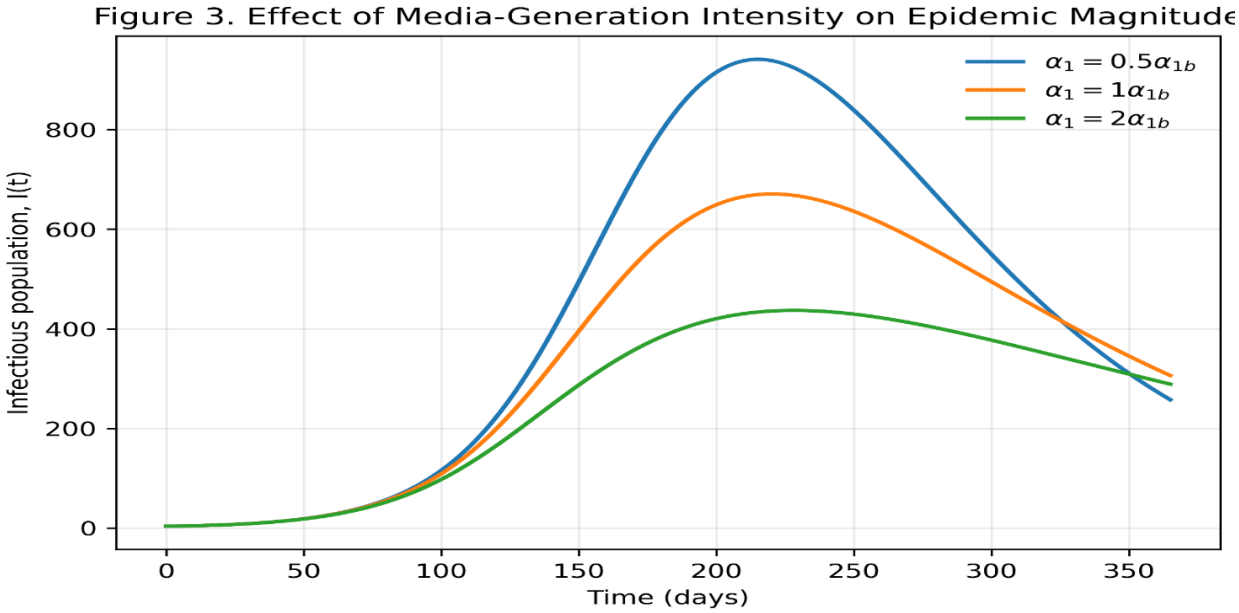
The baseline simulation indicates epidemic persistence because the calculated reproduction number ($R_0 = 1.7157$) exceeds unity. Consequently, the infectious population increased from the initial outbreak level and reached a peak of approximately 4464 individuals, while quarantine and recovery compartments increased as infected individuals progressed through the control pathway. This behaviour agrees with the analytical result that the disease-free equilibrium is unstable when $R_0 > 1$.

Figure 2: Effect of media awareness on infection prevalence



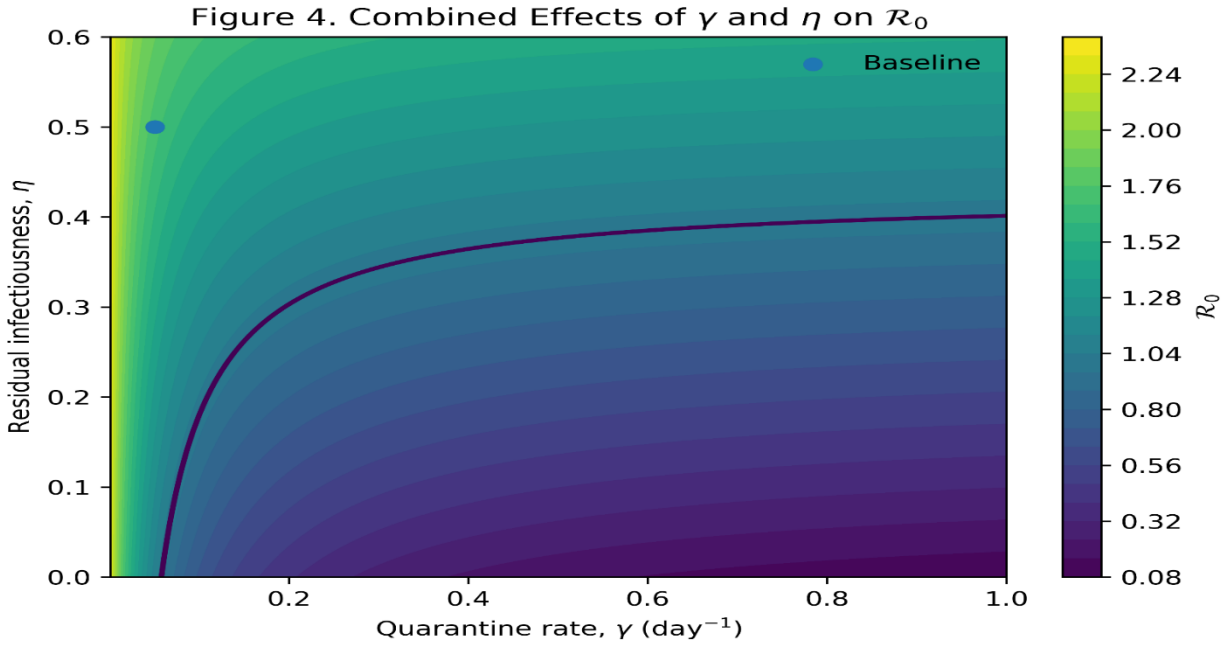
The media awareness had significant impact on the severity of the epidemic. The infectious population was about 1726.30 without media awareness, while the inclusion of media awareness reduced the peak of the infectious population to 670.88, a 61.14% decrease. Moreover, it was found that the cumulative infections reduced from 19618.10 to 12321.56, which amounted to a reduction of 37.19%. The results suggest that awareness primarily affects the size of an epidemic outbreak, but does not influence the invasion threshold

Figure 3: Effect of media-generation intensity



As the intensity of the epidemic media increased, there was a progressive decrease in the magnitude of the epidemics. Increased awareness generation lowered the peak of the epidemic and slowed its spread, with higher information response increases in control effectiveness. Greater information response decreases the opportunities for transmission after disease introduction..

Figure 4: Quarantine rate and residual infectiousness interaction



The effectiveness of the quarantine is shown to be highly reliant on how much infectiousness is reduced during quarantine. When the residual infectiousness rose, the critical quarantine rate went up from $0.05769\ day-1$ to $0.10878\ day-1$, reflecting the fact that if the residual infectiousness is increased, then the effort required to achieve epidemic control is increased as well.

Figure 5: Sensitivity analysis

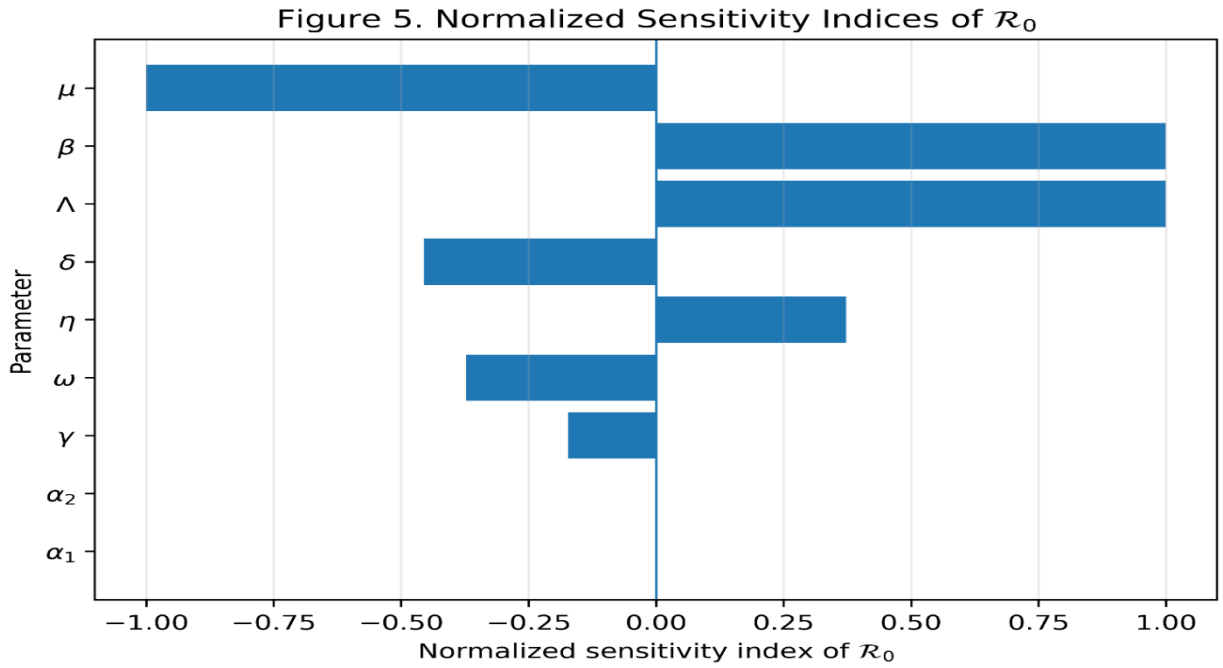


Figure 5 illustrates the normalized sensitivity indices of $R_0$for the model parameters. Positive indices indicate parameters that increase epidemic persistence, while negative indices represent parameters that reduce transmission potential. The parameter ηshows a positive influence, indicating that residual infectiousness during quarantine weakens control effectiveness. The zero sensitivity values of $\alpha_1$and $\alpha_2$occur because $R_0$is evaluated at the disease-free equilibrium, where the media-awareness variable $M(t)$does not appear in the infected subsystem. However, their influence on outbreak progression is captured through the numerical simulations of the full model.

6.Discussion

In this study, an SIQRM-media model was developed to investigate the combined effects of quarantine and media-induced awareness on epidemic dynamics. The framework extends classical compartmental models by incorporating behavioural responses associated with information dissemination and intervention measures. Previous awareness-based studies have demonstrated that information dynamics can influence preventive behaviour and transmission pathways (Misra et al., 2011; Samanta et al., 2013), providing the basis for incorporating media awareness into epidemic models.

The analytical results indicate that the basic reproduction number of the proposed model is $R_0 = 1.7157$, suggesting that the infection can persist under the baseline parameter conditions. This result agrees with established epidemic modelling theory, where control measures must sufficiently reduce transmission below the threshold level to prevent disease invasion (van den

Driessche & Watmough, 2002). Compared with awareness-based models such as Misra et al. (2011) and Samanta et al. (2013), the present model combines awareness dynamics with quarantine processes, allowing the interaction between behavioural response and intervention effectiveness to be examined within a single framework.

The numerical simulations showed that media awareness substantially reduced epidemic severity. Without media feedback, the infectious population reached approximately 1726.30 individuals, whereas incorporating media awareness reduced the peak infection to 670.88 individuals, representing a 61.14% reduction. Cumulative infections were also reduced from 19,618.10 to 12,321.56. These findings support previous studies showing that awareness interventions can modify epidemic outcomes by promoting protective behaviour; however, the magnitude of the reduction depends on model structure, parameter assumptions, and the representation of awareness dynamics.

The quarantine analysis further demonstrated that intervention effectiveness depends on limiting residual infectiousness among quarantined individuals **(Wearing et al., 2005; Wells et al., 2021).** The positive sensitivity of indicates that increased infectiousness during quarantine increases transmission potential and reduces the effectiveness of isolation. This agrees with quarantine-based epidemic models showing that incomplete isolation can weaken disease-control strategies **(Hethcote et al., 2002; Auranen et al., 2023).**

The sensitivity analysis also requires careful interpretation. The zero sensitivity values associated with media parameters occur because the basic reproduction number is calculated at the disease-free equilibrium, where the media-awareness variable does not directly appear in the infected subsystem. Therefore, this does not imply that media awareness has no effect on epidemic dynamics. Rather, its influence is observed during the outbreak progression, as demonstrated by the numerical simulations.

The model has some limitations. The framework assumes homogeneous population mixing and represents media awareness through a simplified dynamic process without explicitly considering misinformation, social network effects, or variations in individual behaviour. Future studies may extend the model by incorporating heterogeneous contact structures, empirical behavioural data, and more detailed information dynamics.

7. Conclusion

This study developed an SIQRM-media framework to examine the interaction between quarantine measures and media-induced awareness in epidemic control. The model demonstrates that behavioural responses and intervention strategies can jointly influence disease dynamics and provides insight into how awareness mechanisms can complement conventional control measures. Although the model is motivated by Dengue transmission, it represents the human behavioural and intervention components of Dengue control rather than the complete mosquito–human transmission cycle. Therefore, the framework should be interpreted as a human-centred epidemic intervention model and not as a replacement for vector-host Dengue models. The findings highlight the importance of considering both epidemiological interventions and behavioural responses when analysing disease-control strategies. Future improvements may incorporate vector dynamics, population heterogeneity, and empirical behavioural data to provide a more comprehensive representation of Dengue transmission.

8.Recommendations

1. Quarantine programmes should maintain effective quarantine rates above the critical threshold identified in this study ($0.05769–0.10878$ day $^{-1}$) to minimize transmission risk.

2. Public health authorities should integrate sustained media-awareness strategies with quarantine measures, as the model demonstrates that awareness mechanisms can enhance epidemic control.
3. Future studies should extend the framework by incorporating Caputo fractional-order derivatives to capture memory effects and non-local dynamics in epidemic transmission processes

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